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questioned by any of us who teach it. Too often, however, we fall into a contented routine of meeting our classes with self-satisfied assurance, assigning the regular lists of problems, conscientiously explaining to the students how to surmount their difficulties with Problem 7 or Formula 42, and leaving our teaching dead with lack of emphasis and discrimination. Not seldom we fail to point out the beauties of the forest because we ourselves see only the trees. To bring us back to fundamentals such an article as Professor Rietz's discussion below must be welcomed by every earnest minded teacher of the subject.

The second discussion, by Professor Bennett, points out the disadvantages attendant on each of the usual conventions for attaching an algebraic sign to the distance between two points or between a point and a line, in analytic geometry. He concludes that the plan most commonly found in our text-books (which he calls the transcendental determination) presents more serious difficulties than either of the others. We are glad to endorse his request for "a reply in defense of the accepted methods."

As the last discussion, we present an interesting little study by Mr. B. H. Brown on a subject which has probably not hitherto received so detailed a mathematical treatment. The close agreement between theory and practice in the probabilities of various occurrences connected with "crap-shooting" is surprising. Some of the questions discussed may furnish suitable material for class problems, as for example the verification of the figures in the columns marked "Theoretical" at the beginning of the article. Devotees of the game will scarcely find in the article arguments to support its availability as a rapid road to riches.

I. On the Teaching of the First Course in Calculus.[1]

By H. L. Rietz, University of Iowa.

In his presidential address[2] read before the American Mathematical Society, Osgood expressed the view that "the calculus is the greatest aid we have to the appreciation of physical truth in the broadest sense of the word." This fact should serve to keep before teachers of the first course in calculus the following important question: Can we teach this course so that our students will feel more keenly than at present the use of the calculus in the appreciation of physical truth?

One needs only compare the text-books in calculus at present with those in use thirty years ago to be convinced that much progress has been made along this line. The first course in calculus has been enriched through a broad range of problems from everyday experience and from physical science. Moreover, logical flaws centering around the conceptions of the infinitesimal and differential have been so largely removed that the mental satisfaction derived from a close contact with physical problems has been much increased. This does not mean that time is given in the first course to rigorous demonstrations, but that time is saved through removing mysticism by a clear statement of definitions and assumptions. It goes without saying that further improvement in the problem work of the course will lead towards the attainment of that object of the calculus expressed in the above quotation, and that all teachers of the subject should devote much thought to new problems for the course. In the present paper we shall say very little about the character of problems, as it is our purpose to emphasize mainly another but related phase of the teaching of the calculus.

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[1] Read before the Iowa Section of the Mathematical Association of America, April 26, 1919.

[2] *Bulletin of the American Mathematical Society*, Vol. 13, p. 449.

It has been a matter of interest to me to test the results of my own teaching of the first course in calculus and those of other teachers by asking students some years out of college, say ten years out of college, about the mental picture they retain of the nature and uses of the calculus. Naturally great variations occur in the picture, depending upon the ability and the life work of the man in question. It has been particularly interesting to me to consider on the one hand, the case of the physicist, the chemist, the actuary, and the engineer; and on the other, that of the man whose life work calls mostly for qualitative rather than quantitative thinking. Some of those who are making no professional or occupational use of the methods of the calculus seem to retain only a general and rather vague notion of certain formulas and transformations applied to the finding of areas and volumes. Indeed, with respect to most of the men who have little quantitative thinking to do the formulas are forgotten. But it has been a source of great satisfaction to find that many of these men long out of college have a picture of the method of the calculus that shows something of how it has entered into their thinking. They still picture the differential calculus as a method which gives us a precise conception of the rate of change of functions and quantities in the phenomenal world. Some have told me that in the integral calculus, they learned to add an infinitely large number of infinitely small quantities. One of my students who took calculus several years ago and who has to make no direct use of the calculus told me recently he could never forget this as fundamental in the calculus and that he appreciates to some extent the importance of this problem because most of the things about us seem to be made up of an infinitely large number of infinitely small parts. Although this conception of adding an infinitely large number of infinitely small parts may savor of an archaic view of the infinitesimal and involve a certain form of mysticism, still I would far rather find that the student has retained such a picture of the calculus than a picture of forgotten formulas.

For our consideration as teachers of the first course in calculus let me submit a brief description of what seems to me should be the minimum of detail in the mental picture retained by the average student ten or more years after studying a first course in the calculus. It seems to me the picture should include at least the following:

(1) That the determination of the rate of change of any quantity is a problem of the differential calculus and that this problem is of almost universal interest in relation to quantitative phenomena.

(2) That by the methods of the differential calculus approximate rates of change of quantities are replaced by exact formulas.

(3) That any limited physical quantity can be represented, with an approximation as close as you please, as a sum of simple and similar component elements, and that by the methods of the integral calculus such approximate sums are replaced by exact formulas.

(4) That physical laws are very generally formulated as differential equations.

The man who has formed a picture, with these notions as the main lines, to aid in the appreciation of the physical world may well be said to have formed "the calculus habit" in his thinking. It is our problem to teach the first course in calculus so as to establish this calculus habit of thought in our students. The important question in this connection is: By what means can we more successfully accomplish this end in our teaching? We would no doubt agree that each of the four points named above should be illustrated many times by concrete problems representing situations in everyday life and in the phenomenal world. We are no doubt doing much at the present time in our teaching of the first course in calculus that tends to carry out the purpose which I have in mind. Thus, we are all accustomed to impressing upon our students the necessity of looking upon a rate of change with respect to time as the limiting value of an average rate of change. We thus replace the approximate rate by an exact conception. We are also no doubt carrying our problems on areas, volumes, masses, moments of inertia, energy, and work back to the central point of looking upon the quantities as limits of the sum of simple and similar component elements. And yet, with all the beautiful and interesting problems that are given and will be given in the first course in the calculus, some of us have a strong feeling that the appreciation of the subject could be increased if more emphasis were placed on the fact that the solution of these problems may well be regarded as interesting details of a picture whose main lines are in a few fundamentals such as the four points mentioned above. The habits of the average student and perhaps even those of the exceptional student are formed largely through repetition. If we would have our students form "the calculus habit" of thinking or would make the calculus such an aid for them as it should be in their appreciation of physical truth, we must recall the main features of our picture whenever we fit into it the various applications to problems of physics, chemistry, geometry, engineering, and everyday life. Otherwise the calculus habit in thinking is not likely to be formed.

In thus contemplating the picture which we would have our students form of the calculus as a fundamental aid in the appreciation of physical truth, it is not to be inferred that the technique or formal side of the calculus may be neglected as unessential. Knowledge of the elements of the calculus is acquired largely through formal work. It is only after attaining considerable proficiency on the algebraic side of the calculus that one is ready to form the picture that should be retained. In this connection it is sometimes maintained that a knowledge of the derivatives and integrals of a few simple functions is adequate preparation in the calculus for an engineer. This position is not without a certain element of truth. It is possible to state the integrals necessary to follow an elementary treatment of physics and mechanics in brief space, but a well-grounded knowledge of the subject that will abide with the student is not to be obtained by confining the algebraic work within very narrow limits. It seems to me that with respect to technique neither the extreme view of algebraic formalism that was in vogue twenty-five years ago in the first course in calculus based on certain English textbooks very generally in use in this country nor the practical lack of formalism advocated by those who would popularize the calculus, makes a satisfactory contribution to our picture of the first course. Surely without approximately the amount of technique given in our recent textbooks we should give totally inadequate preparation for the serious study of mechanics, electricity, thermodynamics, rigid dynamics, hydrodynamics, the theory of elasticity, or for the reading of much of the recent literature of physics and chemistry. Indeed, a second course in calculus has been so strongly urged by certain prominent electrical and mechanical engineers that a course has been introduced into the junior year in certain schools to meet the demand. It goes without saying that technique is indispensable to the student who is to become a mathematician and it is clear that we should take due account of this class of men in looking to the future of education along mathematical lines. It seems to me that some of our recent textbooks in the elements of calculus exhibit rare judgment on the part of the authors in their position with respect to the amount of formal work to be done in the first course in calculus.

In conclusion, let me submit that we can make the first course in calculus a more important aid in the appreciation of physical truth by aiming more directly at establishing in the minds of our students what I have tried to indicate as the calculus habit of thinking about phenomena and that this result is most likely to be obtained by creating for the student a mental picture with only a few main lines.

II. THE SIGN OF THE DISTANCE IN ANALYTICAL GEOMETRY.

By ALBERT A. BENNETT, University of Texas.

The pupil in a course in analytical geometry comes very early across two formulas concerning distances. These yield the following schematic equations:

$$[\text{distance from } (x', y') \text{ to } (x, y)]^2 = (x' - x)^2 + (y' - y)^2,$$

$$[\text{distance from } (x', y') \text{ to } ax + by + c = 0]^2 = \frac{(ax' + by' + c)^2}{a^2 + b^2}, \qquad (a^2 + b^2 \neq 0).$$

No discussion of the derivation of these formulas need be made here. The question of present interest is what is to be done in determining the algebraic sign of the distance itself, that is the square root of each of the above expressions. Now the pupil has become accustomed in dealing with square roots to employ conventions of three types with respect to the determination of signs: